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Jets of nuclear matter in He-A_T inelastic collisions at 4.5 A GeV/c

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Abstract. The problem of the nuclear matter jets in nucleus-nucleus collisions at 4.5 A GeV/c is discussed. The global analysis of experimental data, namely the sphericity tensor, is used to evidence such jets. The experimental results are compared with those obtained in the same collisions for hydrodynamic flow and thrust. The experiments have been performed in the frame of the SKM 200 Collaboration from JINR Dubna.

PACS. 25.70.-z Low and intermediate energy heavy-ion reactions – 25.75.+q Relativistic heavy-ion collisions

1 Global analysis

In the study of the relativistic nuclear collisions dynamics the collision geometry is important [1-6]. The well known participant-spectator picture of these collisions is a strong support for different collision mechanism models, as: intranuclear cascade [7-9], thermal models [10-12], hydrodynamic models [13-17]. All collision mechanism models suppose two important stages, namely: compression stage and expansion stage [4-17]. In the compression stage – which is the first stage after collision – very high densities and temperatures can be obtained in the overlapping region of the two colliding nuclei. Usual physical quantities – density, entropy, thermal energy per baryon, compressional energy per baryon, etc. – do not contain enough information on this stage. They can present, also, a few disadvantages, namely: the achieving of the saturation value in latter stages of the expansion of the fireball, difficulties in the handling of the experimental data or the impossibility to establish experimentally some physical quantities. Therefore, other experimental methods must be found to obtain such information.

One of the method used to find the physical quantities that are sensitive to the compression stage is the *global analysis*. This analysis is related, especially, to the hydrodynamic models [13-17]. The method uses squared combinations of the Cartesian components of the momenta of the particles yield in relativistic nuclear collisions, namely, $p_i \cdot p_j$, combinations which present the advantage of a saturation behavior immediately after the reaching of the highest density at the end of the compression stage [13-24]. Thus, direct information on the behavior of the nuclear matter in the first moments of the collisions can be obtained.

The global quantities can be established by calculation of the global variables for each event, taking into account all particles emitted in an event; in this case the sum total of the kinematic quantities defining the particles from the event is calculated. The components of some tensors are obtained. Subsequently, these tensors are diagonalised, and the diagonal components of the tensors are used to define different interesting physical quantities.

In the global analysis of the nucleus-nucleus collisions at high energies several types of tensors have been proposed [13,14,25,26].

This type of analysis is important to investigate the dynamics of nuclear matter formed in such collisions. In the hydrodynamic description of such collisions instantaneous local equilibrium is required. This equilibrium can be establish if the particles collide sufficiently frequently to be thermalized. As a consequence, a large number of participant nucleons must be involved in collisions for a correct hydrodynamic description. An important result of the assumption of the instantaneous local equilibrium is the fast conversion of the longitudinal momenta in transverse momenta. In this case the side-ward emission of the nuclear matter can be observed [24,25].

The goal of the present work is the discussion of the physical meaning of the different global tensors used in the description of the relativistic nuclear collisions, as well as the presentation of the experimental results on the sphericity tensor – related to nuclear matter jets configuration – in different inelastic He-A_T non-symmetrical collisions at 4.5 A GeV/c. These experimental results are connected with the results obtained using the flow tensor and thrust tensor [18-24]. The experiments have been performed at the Syncrophasotron from the JINR Dubna in the frame of the SKM 200 Collaboration [1,4-6,27-39].

The possible fluctuations and the side-ward flow observed by us in the same collisions [18-24] justify this attempt. In our new analysis we introduce the total cumulative number, $Q = \sum_i Q^i$ [1,35], determined for all negative pions involved in the supposed jet configuration, as a quantitative expression of the local anomality in the nucleon-nucleon phase space. To calculate the cumulative number the following relation has been used:

$$Q^i = E_{iT} \exp(y_i - y_P)/M_N$$

where E_{iT} is the transverse energy of the particle of interest, y_i is the rapidity of the same particle, y_P is the rapidity of the projectile nucleus, and M_N is the rest mass of the nucleon. Our experimental results indicated that the cumulative particle production can be related on the non-equilibrium states of the nuclear matter [40]. The cumulative particles are produced, especially, on transverse direction in comparison with the beam direction. It is important to separate the different collective behaviors determined by equilibrium and non-equilibrium processes.

2 Specific quantities in the global analysis

We mentioned that to do global analysis of the events obtained in relativistic nuclear collisions several forms of global variables and tensors have been proposed [13,14,25, 26,18-24].

The most used tensor is the kinetic flow tensor [13,14, 25]. The components of this tensor are defined by the following relationship:

$$F_{ij} = \sum_n w(n) p_i(n)^{\cdot} p_j(n) \qquad (1)$$

where $i, j = x, y, z$, n is the number of the track, and $w(n)$ is a weight factor of the particle or fragment in the n-th tack. The tensor components form an ellipsoid in the momentum space, and the axes of the ellipsoid are determined by the eigen-values of the tensor. The eigen-values of the tensor are used to define some global quantities, namely: flow ratio – defined like the ratio between the largest axe and the smallest axe of the ellipsoid – and flow angle – considered as the angle between the largest axe of the ellipsoid and the direction of the incident (projectile) nucleus.

For He-A_T collisions at 4.5A GeV/c – using a global flow tensor and a pionic flow tensor – the most important experimental results are the following [18,20,22,24]: (i) The average experimental flow angles are between 80^o and 90^o, and the flow angles obtained by Monte Carlo modelation are around 20^o (Fig. 1.). For inelastic collisions, especially, this behavior is related to the side-splash or squeeze-out phenomena. (ii) The average flow ratio decreases significantly with the increase of the mass number of the target nucleus. (iii) The dependence of the ratio between the number of particles with flow angles higher than 80^o, $N(\theta_f > 80^o)$, and the number of particles with flow angles lower than 80^o on the asymmetry coefficient – $\alpha = (A_T - A_P)/(A_T + A_P)$ – as well as the behavior of interesting physical quantities on different cuts in transverse and longitudinal momentum suggest the influences of the collision geometry in such collisions [28].

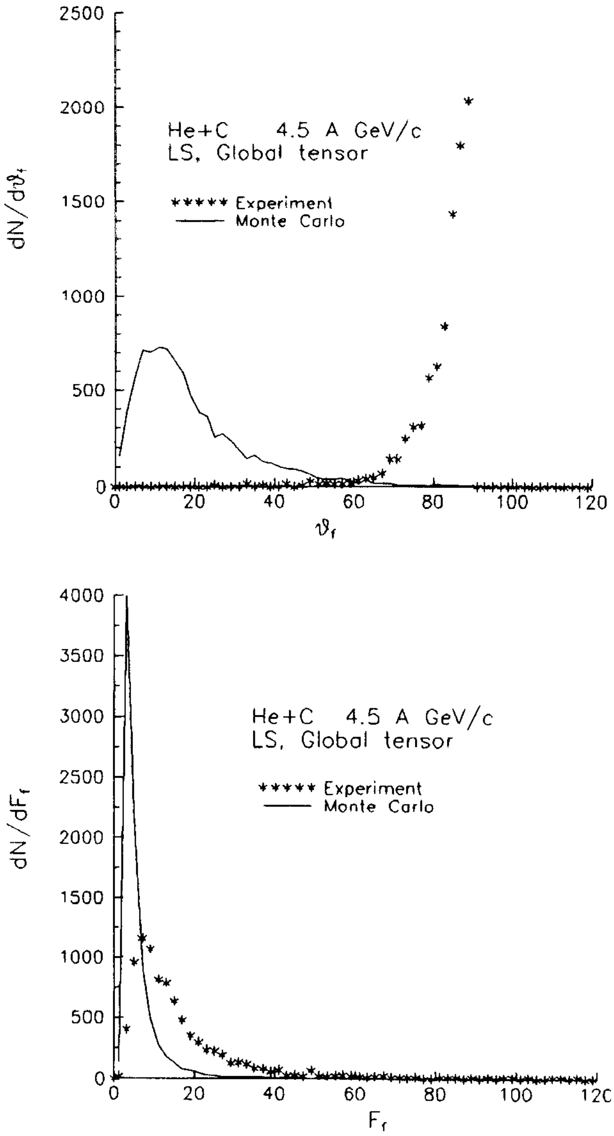

Fig. 1. The flow angle and the flow ratio distributions for global tensor

The most probable direction for the initial flow direction of the nuclear matter from the overlapping region of the two colliding nuclei can be established using the quantity called thrust. This quantity can be defined as follows [23,24,26]:

$$T = \max \frac{\sum_n |\mathbf{p}(n)\mathbf{e}|}{\sum_n |\mathbf{p}(n)|} \quad (2)$$

where $\mathbf{e}$ is the versor of a certain direction. The direction given by $\mathbf{e}$ – for which the maximum value of the thrust is obtained – could be the direction with highest probability for the flow of the very hot and dense nuclear matter formed in the overlapping region of the two colliding nuclei, at intermediate and high energies.

For a few He-A_T collisions at 4.5 A GeV/c a complete analysis of the thrust and direction has been made [23]. The most significant conclusions are: (a) The collision geometry must be taken into account for a complete dynamic description of the nuclear collisions at energies of a few GeV/A. For example, the thrust angle distribution differ for quasi-symmetrical and non-symmetrical collisions. (b) The average values of the thrust angles are comprised in the range 13^o-19^o and increase with the collision centrality and the mass number of the target nucleus. (c) The values of the thrust angles and direction indicated the possibility to have a particle generation in cones on different direction – in agreement with the image on the film. We called these cones as jets of nuclear matter.

The sphericity, an other global quantity of interest, can be calculated using the following tensor:

$$S_{ij} = \sum_n [p^2(n)\delta_{ij} - p_i(n)p_j(n)] \quad (3)$$

Solving the eigen-values and eigen-functions equation it is possible to define the sphericity as:

$$S = (3\lambda_3)/(\sum_{i=1}^{3} \lambda_i) \quad (4)$$

where λ_i are the eigen-values of the tensor (2), decreasing ordered. The sphericity takes values between $S = 0$ - when 2 jets appear, in opposite directions – and $S = 1$ – for isotropic emission.

Using these eigen-values the planarity can be introduced by the following relation:

$$P = [3(\lambda_3 - \lambda_2)]/(\sum_{i=1}^{3} \lambda_i) \quad (5)$$

The form of the flow diagram can offer different arguments for a hydrodynamic behavior of the nuclear matter in nucleus-nucleus collisions at high energies. It is important to stress here that many of these quantities are used in Particle Physics, too, to establish the jet structure of the particle production and the quark structure of the hadrons [41-47]. Therefore, in the following the basic meanings of some quantities are presented.

3 Methods to prove the existence of the nuclear matter jets

The experimental methods to prove the existence of the nuclear matter jets are included in the global analysis of the experimental data.

The first method used the sphericity analysis, introduced first of all in the works [48,49]. In this method, for each event, one searches an axe for which the quantity $\sum_i p_{Ti}^2 (i = 1, n_{ch})$ is minimized, and a quantity, called sphericity, is calculated. The sphericity can be defined by the following equation, too:

$$S = (3\sum_i p_{Ti}^2)/(2\sum_i p_i^2) \quad (6)$$

where p_{Ti} is the transverse momentum of the i-th particle, and p_i is the total momentum of the same particle.

It is important to stress here that the sensitivity of the sphericity increases when one has the possibility to introduce the neutral particles. Unfortunately, these particles are detected with difficulty and only few experiments offer such information.

Another definition of the sphericity is that used in the hydrodynamic models for hadronic interactions at high energies. In this case, for each event, the sphericity tensor, given by eq.(5) is calculated. This tensor can be defined in events *with multiplicity higher than 4*. Its ordered eigenvalues are calculated ($\lambda_1 \geq \lambda_2 \geq \lambda_3$) and the sphericity is defined by the eq.(3).

To establish the bi-jet structure of some events other quantities have been introduced, as the thrust and the spherocity [50,51]. These quantities can be defined be the following relations:

$$T = \frac{\sum_{i=1}^{n} |\mathbf{p}_{Li}|}{\sum_{i=1}^{n} |\mathbf{p}_i|} \quad (7)$$

respectively,

$$S_0 = \left(\frac{4}{\pi}\right)^2 \left(\frac{\sum_{i=1}^{n} |\mathbf{p}_{Ti}|}{\sum_{i=1}^{n} |\mathbf{p}_i|}\right)^2 \quad (8)$$

The eigenvalues of the thrust tensor are between 1/2 and 1. For $T = 1$ all particles have the same direction of the momenta, and for $T = 1/2$ the particle emission is isotropic. The jet axe is chosen to maximize the T value, namely:

$$T = \max(\mathbf{n}) \left(\frac{\sum_{i=1}^{n} |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_{i=1}^{n} |\mathbf{p}_i|}\right) \quad (9)$$

where $\mathbf{n}$ is the versor of the jet axe. In the case of the spherocity the jet axe is chosen to minimize the spherocity.

For events with three jets structure the triplicity is defined [52] as:

$$TR = \max\left(\frac{\sum_{j=1}^{3}\sum_{i\in C_j} \mathbf{p}_i \cdot \mathbf{n}_j}{\sum_{i=1}^{n}|\mathbf{p}_i|}\right) \quad (10)$$

Here, each i particle is included in one of the three class $C_j, j=1,3$, and $i \in C_j$ if the following condition is fulfilled:

$$\mathbf{p}_i \cdot \mathbf{n}_j \geq \mathbf{p}_i \cdot \mathbf{n}_k \qquad \text{with } j \neq k$$

Other quantities to study the events with three jets structure are related to the following variables:

$$\chi = \min\left(\frac{|\mathbf{q}_i|}{\sum_{i=1}^{3}|\mathbf{q}_i|}\right) \quad (11)$$

$$\eta = \max(\cos(\mathbf{n}_i, \mathbf{n}_j)) \quad (12)$$

where $\mathbf{q}_i$ is the total momentum of the particles from the C_i class. The C_i classes, i=1,3, are determined from the triplicity calculation. The variable χ characterizes the momentum distribution in each class C_i, and the variable η describes the orientations of the triplicity axes [53]. To analyze the events with four jets structure the di-thrust tensor has been introduced [54]. This quantity is defined by the following relation:

$$DT = \max\left(\frac{\sum_{j=1}^{2}\sum_{i\in \overline{C}_j} |\mathbf{p}_i...\mathbf{n}_j|}{\sum_{i=1}^{n}|\mathbf{p}_i|}\right) \quad (13)$$

where $\mathbf{n}_1$ and $\mathbf{n}_2$ are the unit vectors on the di-thrust axes. The i particle is included in the class C_i if the following condition is satisfied:

$$|\mathbf{p}_i \cdot \mathbf{n}_1| > |\mathbf{p}_i \cdot \mathbf{n}_2| \quad (14)$$

Two variables must be introduced, namely:

$$\chi' = \min\left(\frac{|\mathbf{q}_i|}{\sum_{i=1}^{3}|\mathbf{q}_i|}\right); \eta' = \max|\cos(\mathbf{n}_1 \cdot \mathbf{n}_2)| \quad (15)$$

To calculate the variables used to describe the jets different methods can be used [55-57].

To study the nuclear matter jets structure of the events for relativistic nuclear collisions two class of methods have been proposed, namely: (i) hierarchic methods; (ii) non-hierarchic methods. In our analysis a non-hierarchic method has been chosen [18,19,21].

The hierarchic methods use the hypothesis that the resemblance among objects (particles) can be described by a measure of the resemblance, called distance. The association of a numeric value to the resemblance involves the possibility of the particles emitted from a given collision to be located in the nodes of a branching structure.

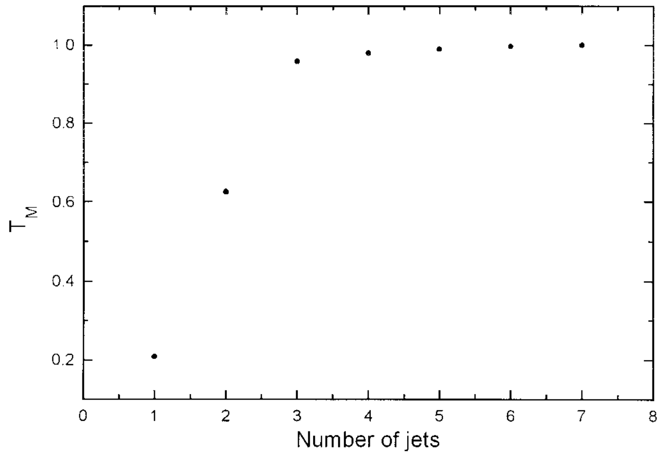

Fig. 2. The dependence of the T_M quantity on the jet number

A non-hierarchic method is the weight center method. The N particles from an event are randomly divided in M groups, with $M < N$. For each subgroup the weight center is calculated. In the second step each particle is included in the subgroup which has the weight center at the shortest distance. For the obtained subgroups after this step/iteration, the new weight center are calculated. The iteration process is stopped when a stable cluster configuration is obtained.

As a quantitative measure of the "distance" between two particles the angle between their momentum vectors can be chosen:

$$\theta_{ij} = \arccos\left(\frac{\mathbf{p}_i \cdot \mathbf{p}_j}{|\mathbf{p}_i| \cdot |\mathbf{p}_j|}\right) \quad (16)$$

To establish if the number of subgroups M for the N particles from an event is equal with the number of jets from this event the following quantity is introduced:

$$T_M = \frac{\sum_{j=1}^{M}|\sum_{i=1}^{m_j}\mathbf{p}_i^j|}{\sum_{l=1}^{N}|\mathbf{p}_l|} \quad (17)$$

This quantity takes values between 0 and 1. The value $T_M = 0$ corresponds to the case when all particles are in a single group, while $T_M = 1$ corresponds to the case when each particle from the event forms its own group. The value $T_M = 0$ can be obtained only in the case when the neutral particles are included.

Taking into account the assumption for the eq.(17) the dependence of the T_M quantity on the jet number is that from Fig. 2. A characteristic behavior for this quantity is the fact that if the value M is the jet number, then T_M discontinuously tends to 1.

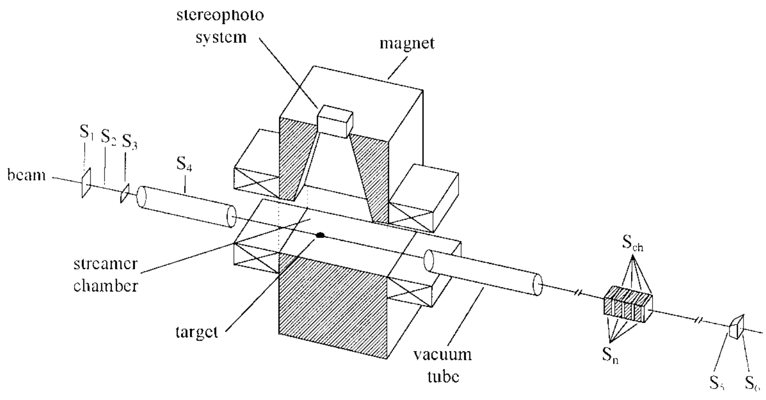

Fig. 3. The experimental set-up from JINR Dubna – The SKM 200 Spectrometer

4 Experimental set-up

The non-hierarchic method presented previously has been used to analyze – for sphericity – different inelastic and central nucleus-nucleus collisions at 4.5 A GeV/c [18-24]. The experiments have been performed at the Syncrophasotron from JINR Dubna (Russia), in the frame of the SKM 200 Collaboration [28-38].

The experimental set-up is presented in Fig. 3. The Syncrophasotron from JINR Dubna can accelerate He, C, O, Ne and Mg heavy ions up to 4.5 A GeV/c, with beams intensities between 10^4 and 10^{12} nuclei/pulse.

The SKM 200 Spectrometer – used by the SKM 200 Collaboration – has a streamer chamber as a main detection device. The chamber is filled with neon at atmospherical pressure and placed in a magnetic field of 0.8 T. Inside the chamber – having the dimensions 2m x 1m x 0.6m – are mounted solid targets in the form of thin discs; in a few experiments the gas that fills the chamber has been used as a target nucleus. A high voltage pulse – 500 kV/pulse, 10.5 ns length of pulse – supplied by a Marx generator is applied on the three electrodes of the streamer chamber. The inferior electrode, in the form of a plate from dural, contains the nine fiducial marks, too.

A stereo-photographic system with 3 cameras – placed in the upper pole of the magnet – allows the recording of the experimental information on high sensibility films.

The streamer chamber can be triggered by two systems of scintillation detectors placed in front and at the back of the streamer chamber. There are two triggering modes: peripheral (inelastic) ($T(\theta_{ch} = 0, \theta_n = 0)$) and central ($T(\theta_{ch} > 0, \theta_n \geq 0)$). θ_{ch} and θ_n are the minimum accepted values of the emission angles for the charged fragments, respectively, neutral fragments of the projectile nucleus with momenta higher than 3.5 GeV/c per particle (stripping fragments).

The experimental data have been obtained through scanning, measuring and geometrical reconstruction. For the He-A_T collisions considered in this work the absolute error in angle is around 2.9^o for all angles, and the relative error in momentum is around 8% for all the range [4,5,57].

The SKM 200 Spectrometer is a 4π detection device. All charged particles and a few neutral particles which decay in charged particles in the streamer chamber of the spectrometer are recorded on high sensitivity films. For central collisions only few particles are rejected, namely, the stripping particles. In this work only peripheral (inelastic) collisions are considered.

From the pictures recorded on films can be identified directly, with small experimental errors, only the negative pions. Their identification is related to the deviation in the magnetic field and the ionisation degree [4-6,23,27-33]. Participants and a few neutral particles which decay in the streamer chamber can be identified, too, after scanning, measure, geometrical reconstruction, kinematic fit and physical interpretation [4-6,35-38,58-60]. To obtain more experimental information for some interesting dynamic aspects an identification method for the charged particles stopped in the streamer chamber of the SKM 200 Spectrometer has been proposed [5,6,58].

The corrections on the experimental values of different interesting physical quantities – especially negative pion multiplicity and momentum – have been made according the papers [28,29,5,6].

5 Experimental results and discussion

Because different physical phenomena in hot and dense nuclear matter affect and modify the momentum spectra of the negative pions [1-3,5,6,11,14-17,33,61-64] we made some cuts. The most important is the cut in momentum. We considered only the negative pions with momentum higher than 50 MeV/c. Therefore, the possible contribution of the negative leptons – especially muons – is rejected.

The stopping of the projectile nucleus in the target nucleus is almost complete at this energy [39]. The observed experimental values of the different physical quantities reflect the collision geometry and take into account the absorption in the spectator region for this energy [4,5,61, 65,66].

In the performed analysis the negative pion component obtained in four peripheral (inelastic) collisions – He-Li, He-Al, He-Cu and He-Pb – is used. For these collisions we observed a significant hydrodynamic behavior [18-24]. This behavior has been evidenced using the global analysis. Different behaviors of the flow ratios, flow angles and thrust angles in central and peripheral nucleus-nucleus

Table 1. The experimental values of different interesting physical quantities obtained in inelastic and central He-A_T collisions at 4.5 A GeV/c

$A_P - A_T$	σ_{in}[mb]	n_-^{in}	P_T^{in}	p_T^{in}[MeV/c]	T_{in}[MeV]	σ_{cen}[mb]	P_T^{cen}	p_T^{cen}[MeV/c]	T_{cen}[MeV]
He-Li	327 ± 20	1.02 ± 0.06	4.0 ± 1.0	241 ± 3	69.0 ± 2.0	120 ± 12	5.6 ± 0.6	198 ± 3	98.6 ± 2.4
He-Al	720± 30	1.72 ±0.12	8.1 ±1.1	229 ± 4	77.0 ±2.5	296±23	11.8 ±1.7	188 ± 5	105.8 ±5.0
He-Cu	1150 ±50	2.15 ±0.10	12.5 ±1.1	227 ± 6	78.0 ±4.0	663 ±50	18.0 ±1.1	186 ± 6	107.6 ±4.9
He-Pb	2400±170	2.23 ±0.08	24.9 ±2.5	204 ± 4	94.0 ±3.0	1840 ±160	37.1 ±2.6	167 ± 4	123.6 ±3.7

collisions at 4.5 A GeV/c have been observed [18-24]. For example, the flow ratio in central collisions decreases in comparisons with its value in peripheral collisions. The number of participants per event or per collisions has a influence on the hydrodynamic flow, too. For a given collision the distribution of the flow angle on the participant number is a constant, and the distribution of the flow ratio shows a significant decrease with the increase of the number of participants. For a certain value of the number of participants per event – 20 for many asymmetric collisions – the distribution of the number of participants is constant, in the limit of experimental errors [24]. The thrust angle decreases with the increase of the asymmetry between the two colliding nuclei [23]. All experimental results confirm the important role of the collision geometry and, as a consequence, of the participants in these collisions [4-6,18-24,36,60]. The number of participants in these collisions can be established using the following relation [4-6,36,60]:

$$P = n_{ch} - 2n_{\pi^-} - (n_{sl} + n_r^+ + n_R^+ + n_{p<p_F}) \qquad (18)$$

with n_{sl} the number of particles with momenta p >3.5 GeV/c generated in the angular range according to the triggering mode, $T(\theta_{ch}, \theta_n)$, of the streamer chamber (spectators of the projectile nucleus), n_r^+ the number of positive charged fragments, with high ionization, which have the length of the track chord smaller than a value r, n_R^+ the number of positive charged fragments, with high ionization, having the length of the track chord between the value r and the value R, with $r < R$, $n_{p<p_F}$ the number of the positive charged fragments, with high ionization, which come out from the chamber and have the momenta smaller than the Fermi momentum, p_F. The quantities r, R and p_F depend on the kinematic conditions, the technical performances of the spectrometer and of the measuring devices. For these collisions the values of these quantities are: r=9.24 cm, R=12.58 cm and p_F=240 MeV/c.

The choice of the sphericity as variable is related to the fact that this quantity reflects the form of the events, and new information on the particle production mechanism can be obtained in the hypothesis that the event topology reflect the collision dynamics. Using the eq.(2) and the ordered eigenvalues of the tensor from the equation (3) – $\lambda_1 \geq \lambda_2 \geq \lambda_3$ – we can obtain an ellipsoid with axes given by eigenvalues. The direction of the axe associated with the greatest eigenvalue λ_1 is the "flow direction", namely the direction for which the sum of the transverse momenta is minimized. If S tends to 0 the ellipsoid tends to a line, and if $S = 1$ then the ellipsoid becomes a sphere.

With this variable and the previously presented non-hierarchic method we analyzed different He-A_T inelastic collisions at 4.5 GeV/c. The variables defined in subsections II and III allow to classify the negative pions contained in each event in two and three nuclear matter jets arising longitudinally ("longitudinal jets") or transversally ("transverse jets"), in forward, respectively, backward directions. Each negative pion jet was associated with the total cumulative number. In this mode the possibility that the jet configuration could be a consequence of a collision with a multi-nucleonic target (flucton) can be investigated [1,40].

The inelastic cross sections for He-A_T collisions at 4.5 A GeV/c are included in Table 1 [5,6,58]. In this table are included, too, other interesting physical quantities – multiplicity, participant nucleons, transverse momentum, temperature – for inelastic and central collisions at the same energy [4-6,32,34-39,58-60,67]. Significant increases of the experimental values for all quantities in central collisions in comparison with inelastic (peripheral) collisions are observed. It is important to stress here the different dependence of the temperature on the participant for central and peripheral collisions, namely: linear – for central collisions, respectively, like-saturation, for peripheral collisions [67]. This behavior is in agreement with the observed behavior in hydrodynamic flow, mentioned previously.

The main experimental results on sphericity tensor and nuclear matter jets are the following.

In He-Li inelastic collisions 4026 events have been analyzed. In this collision the main experimental results are the following: average selected charged particles multiplicity $< n_{ch} >= 5.7 \pm 0.2$, average negative particle multiplicity $< n_- >= 1.87 \pm 0.04$, average number of participant protons $< P >= 2.0 \pm 0.5$, average number of participant nucleons $< P_N >= 4.0 \pm 1.0$, inelastic cross section $\sigma_{in} = 327 \pm 20$ mb, average transverse momentum $< p_T >= 241 \pm 3$ MeV/c, average rapidity in center of mass system $< y >$ = -1.20 [15-20,22,58].

From the 4026 events only 2336 events have been chosen, *because they respect the condition imposed for multiplicity, namely: n >4*. In the chosen events the charged particle multiplicities are between 4 and 14. For each event the tensor given in the (2) has been constructed and the sphericity per event has been calculated. A selection after the charged particle multiplicity per event has been made. In Fig. 4.a-d. the sphericity distributions in events with $n_{ch} = 4, n_{ch} = 6, n_{ch} = 8$ and $n_{ch} \geq 9$ are presented. The average values of the sphericity in these cases are the following: $< S(4) >= 0.158 \pm 0.007$, $< S(6) >= 0.223 \pm$

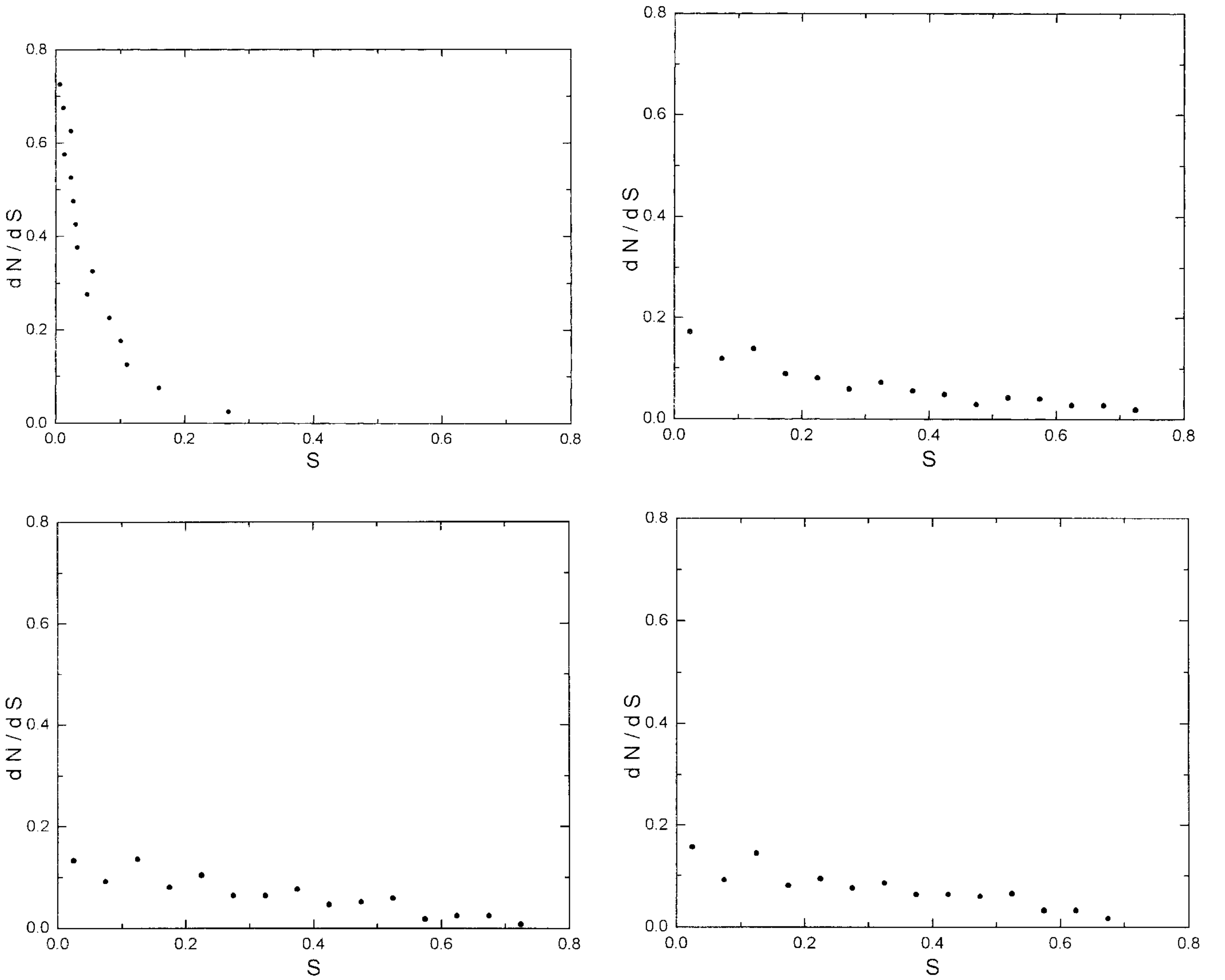

Fig. 4. The sphericity distributions in He-Li collisions at 4.5 A GeV/c in events with: **a** $n_{ch} = 4$; **b** $n_{ch} = 6$; **c** $n_{ch} = 8$; **d** $n_{\mathrm{ch}} \geq 9$

0.09, $< S(8) >= 0.242 \pm 0.008$, respectively, $< S((9) > = 0.273 \pm 0.011$. Such a behavior of the sphericity can suggest a multi-jet structure. For the events with small charged particle multiplicities, a simple superposition of independent nucleon-nucleon collisions can be supposed. For events with higher charged particle multiplicities one can suppose complex particle production mechanisms, with small probabilities. A cumulative production mechanism can be taken into account [1,5,6,35,40].

It is important to stress that the momentum distribution of the quark-antiquark pair must be similar with the emitted hadrons momentum distribution. In this case, in the absence of the correlations between non-consecutive hadrons the following relationship between average sphericity for a given charged particle multiplicity and the corresponding charged particle multiplicity can be written:

$$< S >_{n_{ch}} = (n_{ch}/\alpha) - 1/(\exp(\alpha/n_{ch}) - 1) \qquad (19)$$

where α is a constant. For $n_{ch}/\alpha \ll 1$ the relation (19) can be written as:

$$< S >_{n_{ch}} = n_{ch}/\alpha \qquad (20)$$

In Fig. 5 the distribution of the average sphericity versus the charged particle multiplicity is presented. For $n_{ch} > 10$ the behavior of the dependence changes significantly. This behavior suggests different particle production mechanisms [1-15]. The linear fit with (20) is included in the figure.

An important problem is that of the number of events with two or more nuclear jets. From the 2336 selected events 1433 events have 2 jets, and 903 events have 3 jets. The number of events with 4 jets is 0.

The behavior of the average sphericity for a given charged particle multiplicity on the corresponding charged particle multiplicity suggests the possibility of some cumulative mechanisms in the charged particle production [1,19-24,40]. An analysis for cumulative number [19,21] has been made for the events with 2 and 3 jets.

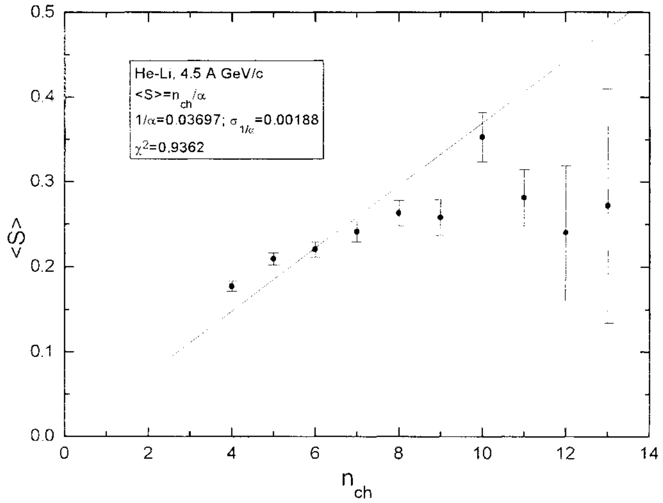

Fig. 5. The dependence of the average sphericity on the charged particle multiplicities in He-Li inelastic collisions at 4.5 A GeV/c

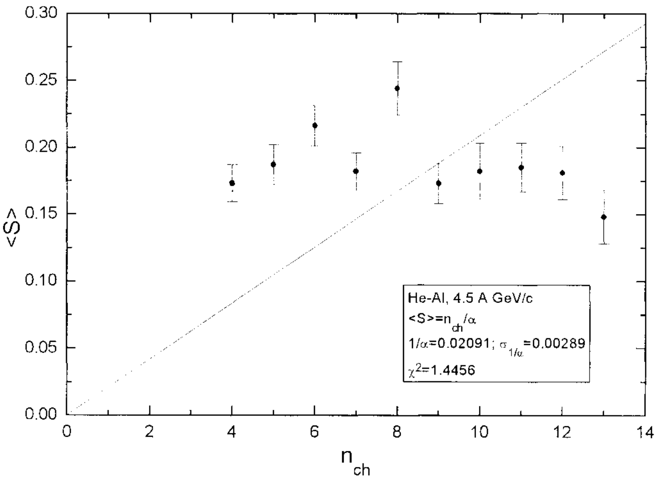

Fig. 6. The dependence of the average sphericity on the charged particle multiplicities in He-Al inelastic collisions at 4.5 A GeV/c

In the 2 jets events the average cumulative number of the particle from the forward jet is $< Q_{2f} >= 1.30 \pm 0.21$, and the average cumulative number of the particle from the backward jet is $< Q_{2b} >= 9.09 \pm 0.99$. The average cumulative number for all events with 2 jets is $< Q_{2T} >= 10.40 \pm 1.21$. These values of the average cumulative numbers are related to the collision kinematics and they are correlated with the jet angles, namely: $\Phi_{2f} = 47.3^o \pm 2.1^o$, respectively, $\Phi_{2b} = 53.1^o \pm 2.7^o$, with $\Phi_{2f} < \Phi_{2b}$.

For 3 jets events the average cumulative numbers in the three types of jets – forward (f), backward (b) and transverse (t) – have the following values: $< Q_{3f} >= 1.05 \pm 0.19$, $< Q_{3b} >= 9.47 \pm 1.13$, respectively, $< Q_{3t} >= 3.20 \pm 0.72$. In this case the average cumulative number for all events with 3 jets is $< Q_{3T} >= 13.72 \pm 1.35$.

Comparing the values of the average cumulative numbers for all events with 2, respectively, 3 jets we can observe that $< Q_{3T} >>< Q_{2T} >$ and the fact that $< Q_{3f} > + < Q_{3b} > = < Q_{2T} >$ the contribution of the transverse jet in the 3 jets events being evident.

In the three jets events, in He-Li inelastic collisions, the average opening angles are the following: (i) forward jet: $< \Phi_{3f} >= 41.9^o \pm 2.5^o$; (ii) backward jet: $< \Phi_{3b} >= 44.7^o \pm 2.6^o$; (iii) transverse jet: $< \Phi_{3t} >= 37.4^o \pm 2.5^o$.

It is interesting to analyze correlations between two physical quantities characterizing the events, such as jet angle and multiplicity, jet angle and cumulative number, transverse momentum and cumulative number etc. For example, if we suppose a linear dependence between the average opening angle and the average cumulative number for the three jets – forward, backward and transverse – the correlation coefficient, r, has the following values: $r_{3f} = 0.143, r_{3b} = 0.179, r_{3t} = 0.127$. A very weak correlation between the two quantities can be observed.

Other three collisions – He-Al, He-Cu and He-Pb – have been analyzed for the same dependencies. Similar behaviors for the sphericity distributions in events with a given charged particle multiplicity and for the average sphericity for a given charged particle can be observed (Figs.6-8.). If we use an other kind of linear fit ($y(x) = a + bx$) to the experimental data then a negative slope is obtained in two of the three collisions. If one uses an exponential growth function for fitting, then the average sphericity in all three collisions can be described with the same function $y(x) = a + b \cdot e^{c \cdot x}$. The fits with this function and the values for a, b and c parameters are included in Figs. 9-12.

In He-Pb collisions from the 1048 events 1004 events have $n_{ch} \geq 4$. From these 1004 events 322 events have three jets. The average cumulative numbers in the three types of jets are the following: $< Q_{3f} >= 1.38 \pm 0.16$, $< Q_{3b} >= 13.77 \pm 2.18$, respectively, $< Q_{3t} >= 2.43 \pm 0.35$. Comparing these values with those for He-Li inelastic collisions at the same energy we observe that only the average cumulative number for backward jet increases significantly. This behavior can be related to the increase of the participants in He-Pb inelastic collisions, as well as on the preferential emission of the cumulative negative pions in the backward side [35,40]. The average number of participant protons, respectively, the average number of participant nucleons in O-Pb inelastic collisions at 4.5 A GeV/c are: $< P >_{HePb} = 7.9 \pm 1.2$, respectively, $< P_N >_{HePb} = 25.0 \pm 2.5$ [4-6,32,36,60]. The inelastic cross section is much higher, too, and average transverse momentum is smaller, namely: $\sigma_{in}^{HePb} = 2400 \pm 170$ mb, respectively, $< p_T >_{HePb} = 204 \pm 4$ MeV/c [4-6,58,60,67].

The average opening angles of the three jets are: $< \Phi_{3f}(HePb) >= 53.8^o \pm 1.7^o$, $< \Phi_{3b}(HePb) >= 54.7^o \pm 1.8^o$, $< \Phi_{3t}(HePb) >= 45.3^o \pm 2.4^o$. In this case we can observe the trend of the forward and backward jets to have the same opening angle. The average opening angle of the transverse jet remains smaller than the average opening angles of the two jets, but is higher than the corresponding angle for He-Li inelastic collisions at the same energy. An increase of the thermalization degree can be expected

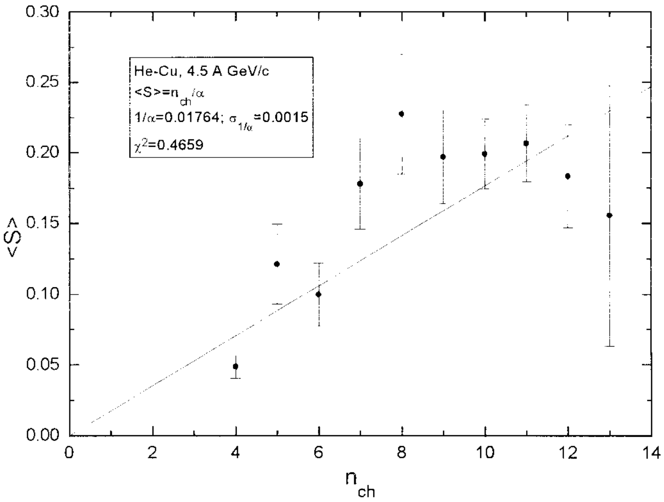

Fig. 7. The dependence of the average sphericity on the charged particle multiplicities in He-Cu inelastic collisions at 4.5 A GeV/c

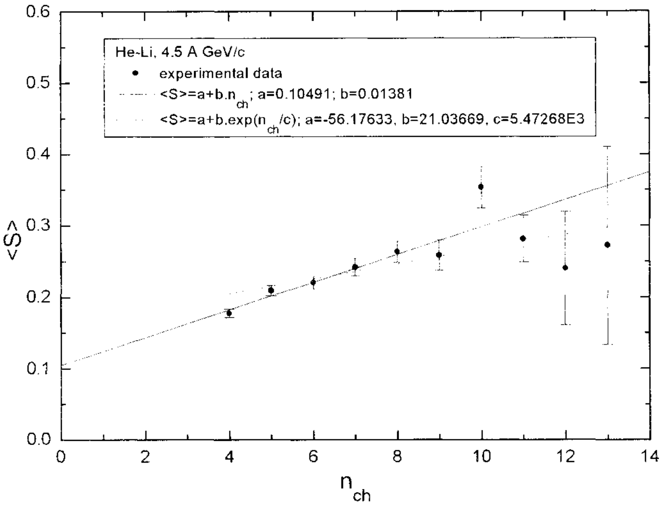

Fig. 9. The dependence of the average sphericity on the charged particle multiplicities in He-Li inelastic collisions at 4.5 A GeV/c – fit with linear and exponential function

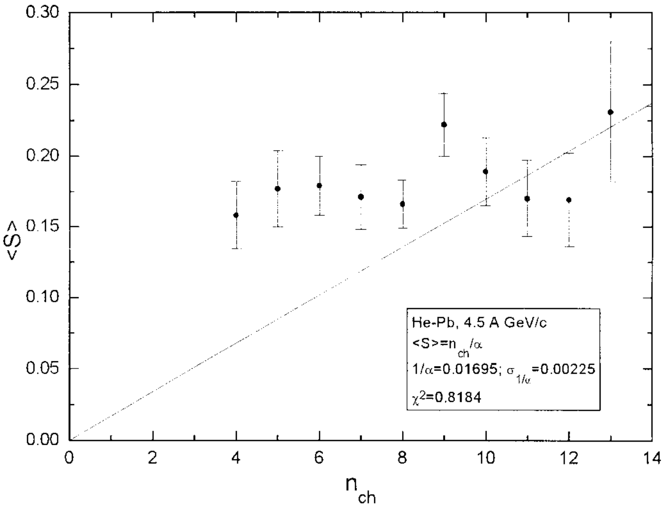

Fig. 8. The dependence of the average sphericity on the charged particle multiplicities in He-Pb inelastic collisions at 4.5 A GeV/c

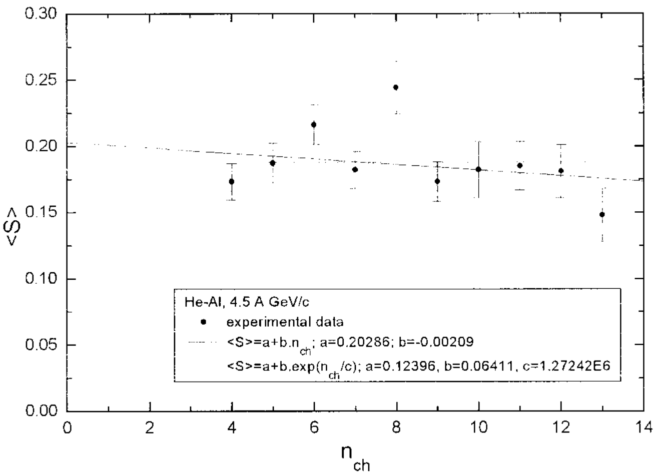

Fig. 10. The dependence of the average sphericity on the charged particle multiplicities in He-Al inelastic collisions at 4.5 A GeV/c – fit with linear and exponential function

[4-6,36,59,67] and, as a consequence, an increase of the opening angle.

The correlation coefficients for a linear dependence of the opening angle on the cumulative number for the three jets are the following: $r_{3f}^{HePb} = 0.148$, $r_{3b}^{HePb} = 0.253$ and $r_{3t}^{HePb} = 0.141$. Comparing with the experimental results for He-Li inelastic collisions we observe the increase of the correlation coefficients for backward and transverse jets. This behavior can be related to the cumulative numbers behaviors, as well as on the collision geometry and collision symmetry. Also, this result can be related to the sideward splash observed in the same collisions in a flow tensor analysis of the hydrodynamic behavior [20,22,24].

These behaviors are present in the three mentioned inelastic collisions. The values of the interesting physical quantities increase with the mass number of the target nucleus.

To obtain more information we calculate the average total momentum for the events from each type of jet, as well as the transverse momentum. In He-Pb inelastic collisions the average values of the total momenta – considered as the sum of the total momenta for all particles from the jet – are the following: $< p_{3f}(He - Pb) >= 6.874 \pm 0.741 GeV/c$, $< p_{3b}(He - Pb) >= 4.041 \pm 0.357$ GeV/c and $< p_{3t}(He - Pb) >= 4.275 \pm 0.456$ GeV/c. Supposing a linear dependence between the opening angle and the average total momentum in each jet we obtain small values of the correlation coefficients for forward and backward jets (0.127, respectively, 0.139), while for the transverse jet the correlation coefficient is near 0.

The Table 2 contains a comparison between the experimental values of the percentage of jets established by the

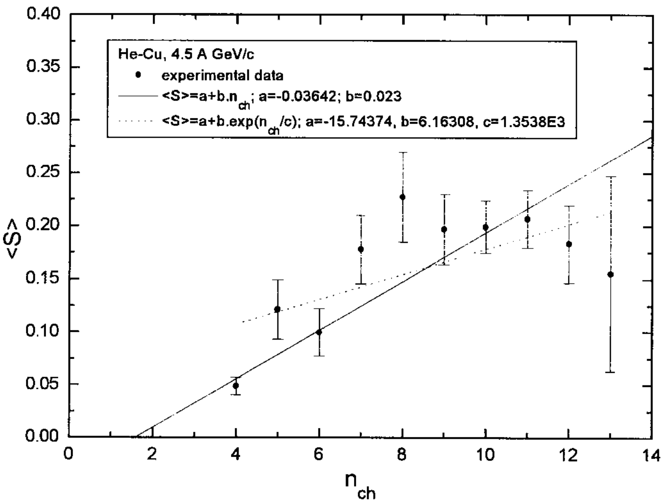

Fig. 11. The dependence of the average sphericity on the charged particle multiplicities in He-Cu inelastic collisions at 4.5 A GeV/c – fit with linear and exponential function

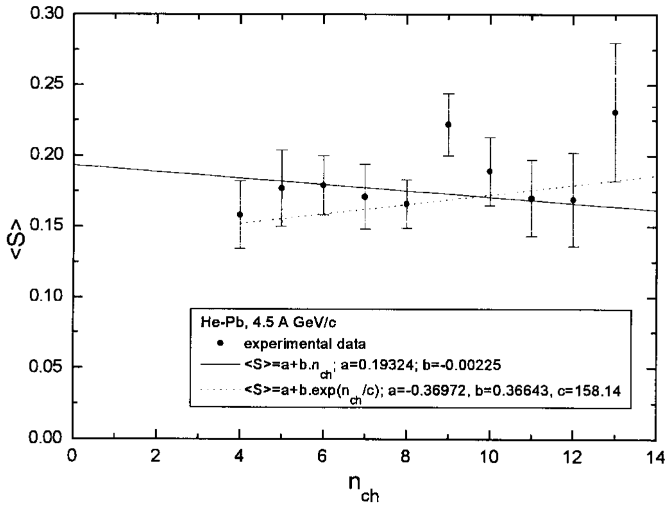

Fig. 12. The dependence of the average sphericity on the charged particle multiplicities in He-Pb inelastic collisions at 4.5 A GeV/c – fit with linear and exponential function

Table 2. Comparison between the experimental and the Monte Carlo evaluation values for the percentage of the jet events in He-Pb collisions at 4.5 A GeV/c

Number of jets per event	P_{exp}	P_{jet}^{ran}
1 jet	0.256±0.030	0.012±0.001
2 jets	0.145±0.040	0.045±0.003
3 jets	0.080±0.007	0.030±0.001

method described previously and the Monte Carlo evaluation of the jet numbers in He-Pb peripheral collisions at 4.5 A GeV/c. In the Monte Carlo evaluation the thrust values were constructed using the experimental multiplicities and experimental momenta for each event. The Cartesian components of the momentum for each particle has been calculated using a random number generator. The percentage R_{jet}^{ran} has been calculated as the ratio between the number of events with $T > 0.9$ and the number of the events randomly constructed. The differences over the limit of the experimental errors are obviously for the three situations considered.

These results confirm the fact that the hydrodynamic behavior of the nuclear matter formed in nucleus-nucleus collisions at 4.5 A GeV/c – including the nuclear matter jets structure – is a physical effect, not a statistical effect (fluctuations).

Finally, we can say that there are some jets of nuclear matter in nucleus-nucleus collisions. The origin of these nuclear jets is different, especially for transverse jets [18-24,40,66,68-71]. For our case, to the transverse jet an important contribution is related to the cumulative generation of particles. It is evident that the nuclear matter jets can be related on different mechanisms [40,60-71]. The collision geometry has an important influence on the jet structure and jet properties, too [4-6,66,70,71]. For example, the transverse jet can be originated in a local equilibrium process, like the hydrodynamic flow, but, also, in a non-equilibrium process, like the cumulative generation. It is important to study more collisions, at different energies to obtain more and more information on the dynamics of the nuclear co llisions at a few GeV/A. In this case the contributions of different effects – non-stopping of the incident nucleus in the target nucleus, equilibrium processes and non-equilibrium process – can be separated.

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